ON THE ELECTROSTATIC DEFLECTION IN A CATHODE RAY TUBE.

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(1) In the ordinary Thomson cathode ray tube for determining the value of ${}^{e}/_{m}$ for cathode rays we have to allow for the irregularity of the electrostatic field near the edges of the charged plates. Sir T. T. Thomson,* in determining the value of ${}^{e}/_{m}$ for rays of positive electricity, has given the solution for the case when the rays are parallel to the plates as they enter between them. It is therefore interesting to find the electrostatic deflection given by the complete solution when the rays are projected parallel to the plates but at some distance from them. The following note is written with the object of drawing attention to the method of the calculation, and of comparing the deflection given by the complete solution with that given on the assumption of a uniform field.

Two check calculations are added to test the accuracy of the method of calculation.

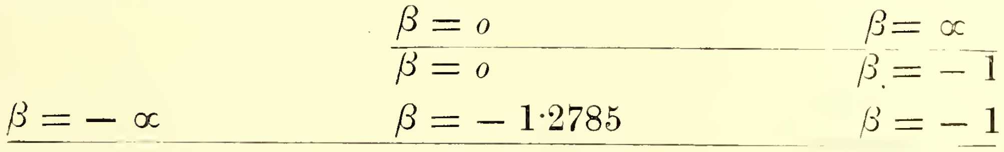
The dimensions of the tube given are those of a tube used at Rhodes University College, Grahamstown.

(2) *Parallel Plate Condenser.*

The complete solution of the semi-infinite parallel plate condenser is given by the Schwarzian transformation—

$$z = \mathrm{C}(\beta - \log \overline{1 + \beta} + i\pi) \quad . \quad . \quad . \quad . \quad (1),$$

where $z = x + iy$. This transforms the infinite plate $y = o$, and the semi-infinite plate $y = \mathrm{C}\pi$ into the real axis of β. The range of β is shown in the following figure:



The further transformation,

$$w = \mathrm{D} \log \overline{1 + \beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

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* 'Phil. Mag.,' S. 6, vol. xx, No. 118, Oct., 1910.

where $w = \mathrm{U} + i\mathrm{V}$, U being the current function, and V, the potential function, gives the solution when the plate $y = \mathrm{C}\pi$ is at potential o, and the plate $y = o$, is at potential $\mathrm{D}\pi$.

The elimination of β between (1) and (2) gives—

$$x = \mathrm{C}\left(e^{\mathrm{U}/\mathrm{D}} \cos {}^{\mathrm{V}}/_{\mathrm{D}} - 1 - {}^{\mathrm{U}}/_{\mathrm{D}}\right) \quad . \quad . \quad . \quad (3),$$

and

$$y = \mathrm{C}\left(e^{\mathrm{U}/\mathrm{D}} \sin {}^{\mathrm{V}}/_{\mathrm{D}} - {}^{\mathrm{V}}/_{\mathrm{D}} + \pi\right) \quad . \quad . \quad . \quad (4),$$

when $\mathrm{V} = o,\ y = \mathrm{C}\pi,$

and when $\mathrm{V} = \mathrm{D}\pi,\ y = o$

Now, by the principle of images, the distribution on the upper plate is the same as if the lower plate were a semi-infinite plate, $y = -\mathrm{C}\pi$, and the differences of potential between them $2\mathrm{D}\pi$.

(3) *Electrostatic Deflection of the Cathode Ray.*

If a particle of mass on carrying a negative charge is projected from a point on the axis of x with velocity v parallel to x, the equation of motion is—

$$m\frac{d^2y}{dt^2} = -e\mathrm{Y} = e\frac{d\mathrm{V}}{dy} = e\frac{d\mathrm{U}}{dx},$$

or approximately,

$$\frac{mv^2}{e}\frac{d^2y}{dx^2} = \frac{d\mathrm{U}}{dx},$$

$$\frac{mv^2}{e}\frac{dy}{dx} = \mathrm{U} - \mathrm{U}_0,$$

where U_0 is the value of U, where $\frac{dy}{dx} = o$.

$$\frac{mv^2}{e}y = \int \mathrm{U}dx - \mathrm{U}_0\int dx,$$

but $x = \mathrm{C}(\beta - \log \overline{1+\beta})$, and $\mathrm{U} = \mathrm{D}\log\overline{1+\mathrm{B}}$,

therefore—

$$\frac{mv^2}{e}y = \mathrm{CD}\int\frac{\beta}{1+\beta}\log\overline{1+\beta}\,d\beta - \mathrm{U}_0\int dx,$$

$$\frac{mv^2}{e}y = \mathrm{CD}\Big[\overline{1+\beta}\log\overline{1+\beta} - \overline{1-\beta} - \tfrac{1}{2}\log^2\overline{1+\beta}\Big]_{\beta_0}^{\beta} - \mathrm{U}_0\Big[x\Big]_{l_0}^{l}.$$

(4) *Cathode Ray Tube.*

The cathode ray is projected at P parallel to the axis of x, and the electrostatic deflection is produced by the charged plates A and B, which are kept at a constant potential difference. The dimensions of the tube are shown in the figure, the distance between the plates being 0·69 cm.

The stray electrostatic field between P and O_1 deflects the ray before it comes between the plates. A small deflection at O_1 would produce a considerable deflection at the distance Q, even supposing there was no further deflection by the electrostatic field.

Let us first consider the effect of the stray field between P and O_1.

Take O_1 as origin, and O_1Q as the positive direction of x. The value of β at P is given by—

$$-4{\cdot}40 = C\,(\beta - \log - \overline{1 + \beta}), \text{ where } 2\pi C = 0{\cdot}69$$

$$\beta - \log - (1 + \beta) = -40{\cdot}06,$$

which gives $\beta_P = -36{\cdot}50$, and $\log - (1 + \beta_P) = 3{\cdot}57$.

The value of β at the origin is given by—

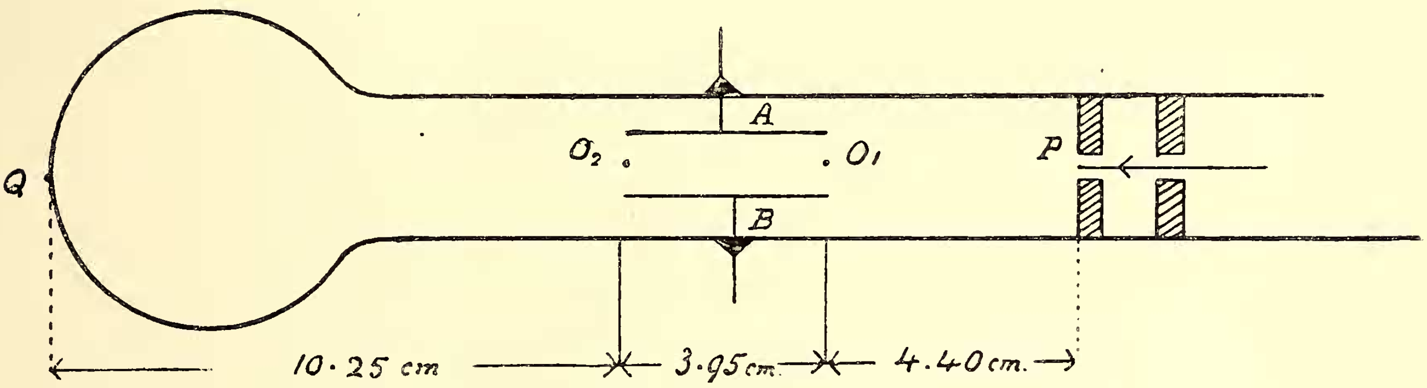
$$\beta_{O_1} = \log - (1 + \beta_{O_1}),$$

$$\beta_{O_1} = -1{\cdot}2785, \text{ and } \log - (1 + \beta_{O_1}) = -1{\cdot}2785.$$

The deflection y_1 at O_1 is given by—

$$\frac{mv^2}{e} y_1 = CD\;\overline{1 + \beta}\log\overline{1 + \beta} - \overline{1 + \beta} - \tfrac{1}{2}\log^2\overline{1 + \beta}\Big]_P^{O_1} - 4{\cdot}4\log\overline{1 + \beta_P}$$

$$\frac{mv^2}{e} y_1 = 97{\cdot}3\;CD - 15{\cdot}9\;D.$$



The velocity parallel to y is given by—

$$\frac{mv}{e}\frac{dy}{dt} = U_{O_1} - U_P$$

$$= -4{\cdot}85\;D.$$

To find the deflection due to the field from O_1 to Q we now take O_2 as origin and O_2P as the positive direction of x.

The value of β at Q is given by—

$$C\,\{\beta_Q - \log\{(1 + \beta_Q)\}\} = -10{\cdot}25,$$

$$\beta_Q - \log - (1 + \beta_Q) = -93{\cdot}31,$$

which gives $\beta_Q = -88{\cdot}82$, and $\log \overline{1 + \beta_Q} = 4{\cdot}47$.

An approximate value of β at O_1 is -1, say $(-1 - \gamma)$,

then $3{\cdot}95 = C\,(-1 - \gamma - \log \gamma)$,

$$\gamma + \log \gamma = -36{\cdot}96.$$

An approximate solution is $\gamma = e^{-36{\cdot}96}$,

therefore $U_{O_1} = D \log - (1 + \beta_{O_1}) = -36{\cdot}96\;D$.

We may now regard the ray as projected from O_1 with velocities given by

$$\frac{dx}{dt} = -v,$$

and

$$\frac{mv}{e}\frac{dy}{dt} = -4{\cdot}85\;D.$$

The equation of motion is—

$$\frac{m}{e}\frac{d^2y}{dt^2} = \frac{dU}{dx},$$

$$-\frac{mv}{e}\frac{dy}{dt} = \mathrm{U} + \mathrm{A},$$

but $$4{\cdot}85\,\mathrm{D} = \mathrm{U}_{\mathrm{O}_1} + \mathrm{A},$$

therefore $$-\frac{mv}{e}\frac{dy}{dt} = \mathrm{U} - \mathrm{U}_{\mathrm{O}_1} + 4{\cdot}85\,\mathrm{D}.$$

$$\frac{mv^2}{e}y_2 = \mathrm{CD}\left[\overline{1+\beta}\log\overline{1+\beta} - \overline{1+\beta} - \tfrac{1}{2}\log^2\overline{1+\beta}\right]_{\mathrm{O}_1}^{\mathrm{Q}_1} - 41{\cdot}81 \times 14{\cdot}2\,\mathrm{D}$$

$$\frac{mv^2}{e}y_2 = 373{\cdot}3\,\mathrm{CD} - 593{\cdot}5\,\mathrm{D}.$$

The deflection at Q is given by—

$$\frac{mv^2}{e}y = \frac{mv^2}{e}(y_1 + y_2) = 470{\cdot}6\,\mathrm{CD} - 609{\cdot}4\,\mathrm{D}.$$

If X be the electric intensity between the plates—

$$2\pi\mathrm{D} = 0{\cdot}69\,\mathrm{X},$$

and we have seen that $$2\pi\mathrm{C} = 0{\cdot}69,$$

then $$\frac{mv^2}{e}y = -\,61{\cdot}2\,\mathrm{X}$$

(5) *Check Calculations.*

(*a*) Instead of integrating from P to O_1 in the first instance we might have integrated from P to O_2. In this case—

$1 + \beta_{\mathrm{P}} = -\,35{\cdot}50$, $1 + \beta_{\mathrm{O}_2} = 0$, $1 + \beta_{\mathrm{Q}} = -\,87{\cdot}82$,

$\log 1 + \beta_{\mathrm{P}} = 3{\cdot}57$, $\log(1 + \beta_{\mathrm{O}_2}) = -36{\cdot}96$, $\log\overline{1 + \beta_{\mathrm{Q}}} = 4{\cdot}47$.

Then—

$$\frac{mv^2}{e}y_1 = \mathrm{CD}\left[\overline{1+\beta}\log\overline{1+\beta} - \overline{1+\beta} - \tfrac{1}{2}\log^2\overline{1+\beta}\right]_{\mathrm{P}}^{\mathrm{O}_2} - 3{\cdot}57 \times 8{\cdot}35\,\mathrm{D}.$$

and—

$$\frac{mv^2}{e}y_2 = \mathrm{CD}\left[1+\beta\log\overline{1+\beta} - \overline{1+\beta} - \tfrac{1}{2}\log^2\overline{1+\beta}\right]_{\mathrm{O}_2}^{\mathrm{Q}} - 41{\cdot}81 \times 10{\cdot}25\,\mathrm{D}.$$

The total deflection is given by—

$$\frac{mv^2}{e}y = \frac{mv^2}{e}(y_1 + y_2) = -\,905\,\mathrm{CD} - 457{\cdot}1\,\mathrm{D},$$

$$\frac{mv^2}{e}y = -\,61{\cdot}1\,\mathrm{X}.$$

(*b*) Let us integrate from P to a point M midway between O_1 and O_2.

The value of $\log(1 + \beta_{\mathrm{M}})$ is approximately $-\left(\frac{3{\cdot}95}{2\mathrm{C}} - 1\right)$,

therefore— $\log(1 + \beta_{\mathrm{M}}) = -\,18{\cdot}98.$

$(1 + \beta_{\mathrm{P}}) = -35{\cdot}50$, $(1 + \beta_{\mathrm{M}}) = 0$, $1 + \beta_{\mathrm{Q}} = -\,87{\cdot}82$,

$\log 1 + \beta_{\mathrm{P}} = 3{\cdot}57$,. $\log(1 + \beta_{\mathrm{M}}) = -\,18{\cdot}98$, $\log(1 + \beta_{\mathrm{Q}}) = 4{\cdot}47$.

Then—

$$\frac{mv^2}{e} y_1 = \mathrm{CD}\left[\overline{1+\beta} \log \overline{1+\beta} - \overline{1+\beta} - \tfrac{1}{2} \log^2 \overline{1+\beta}\right]_{\mathrm{P}}^{\mathrm{M}} - 3{\cdot}57 \times 6{\cdot}37\,\mathrm{D},$$

and—

$$\frac{mv^2}{e} y_2 = \mathrm{CD}\left[\overline{1+\beta} \log \overline{1+\beta} - \overline{1+\beta} - \tfrac{1}{2} \log^2 \overline{1+\beta}\right]_{\mathrm{M}}^{\mathrm{Q}} - 41{\cdot}53 \times 12{\cdot}22\,\mathrm{D}.$$

The total deflection is given by—

$$\frac{mv^2}{e} y = \frac{mv^2}{e} (y_1 + y_2) = -\,217{\cdot}2\,\mathrm{CD} - 529{\cdot}8\,\mathrm{D},$$

$$\frac{mv^2}{e} y = -\,60{\cdot}8\,\mathrm{X}.$$

The values obtained by the different integrations are in sufficiently good agreement.

(6) *Uniform Field.*

If we had neglected the irregularities of the field and considered the ray to enter between the plates parallel to x, the deflection would have been given by—

$$\frac{mv^2}{e} y = -\,3{\cdot}95(10{\cdot}25 + 1{\cdot}97)\,\mathrm{X},$$

$$= -\,48{\cdot}3\,\mathrm{X},$$

which differs greatly from that given by the complete solution.